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# Final Report on PhD degree for

# Neil J. Fergusson

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**Abstract**

This document represents a final report on the PhD degree work of Neil J. Fergusson. The degree was awarded in August of 1991.

# 1 Dissertation Research

The PhD dissertation research for *The Free and Forced Vibrations of Structures Using the Finite Dynamic Element Method* by Neil J. Fergusson is summarized here.

In addition to an extensive review of the literature on exact and corrective displacement based methods of vibration analysis, a few theorems are proven concerning the various structural matrices involved in such analyses.

In particular, The consistent mass matrix and the quasi-static mass matrix are shown to be equivalent, in the sense that the terms in their respective Taylor expansions are proportional to one another, and that they both lead to the same dynamic stiffness matrix when used with the appropriate stiffness matrix. Furthermore, the calculation of the $2j^{\text{th}}$ order term for either of these, can be accomplished with the knowledge of the $j^{\text{th}}$ order shape function term. In addition, proportionality is shown between the mass and stiffness terms, as is the dependence of each of the terms for any of the four matrices $\mathbf{m}$, $\tilde{\mathbf{m}}$, $\mathbf{k}$ and $\mathbf{d}$, on the boundary values of the shape functions.

Dynamic elements for the axisymmetric bending of annular and circular Kirchoff plates have been calculated in closed form by a *Macsyma* program. The complexity of the resulting expressions produces, however, produces intolerable perturbations in the calculated matrices, when evaluated on a finite precision machine. An alternative given here, involves the replacement of these complicated expressions by their respective Laurent series, thus eliminating the inherent rounding errors. The Laurent series expressions are then used to find natural frequencies of a clamped circular plate. The results demonstrate a marked increase in accuracy as compared to similar calculations made using statically based elements.

Although the illustration given results in a quadratic eigenproblem, the method is easily extended to cubic and higher. (The enclosed *Macsyma* program produces the matrices necessary to construct the cubic eigenproblem, although the numerical results are obtained using only a quadratic truncation.

The dynamic element method, involving the solution of a quadratic eigenproblem for the (high accuracy) modes of a vibrating structure, has been extended to the calculation of the response via modal analysis. The new method differs from the conventional modal analysis in that the modes employed are found using the high accuracy dynamic element method. In addition, the associated modal masses used to normalize the computed modes are evaluated using a dynamic correction term in the mass matrix.

The results for the transverse vibration of a cantilevered beam show that a much greater accuracy is obtained for the response when the dynamic element is used instead of the usual finite element method.

The study considers the effects of coupled thermoelasticity on the Dynamic Element Formulation. A general dynamic element approach has been derived to deal with the coupled theory of three-dimensional coupled thermoelasticity, and this is modified to produce a simple mechanics of materials model for extensional bar systems. The material, which includes the simplifications demonstrated earlier in the thesis, is appended by a *Macsyma* program which computes an arbitrary number of terms in the expansions for the mass, damping-like, stiffness, and dynamic stiffness matrices, all dependent on a complex frequency $\omega$.

The work also considers the numerical calculation of the various structural matrices. The two methods described, based on assumed shape elements, and first order numerical integrators, respectively, are only defined, not demonstrated. Some tests, however, have been run, but not written up, which show that the methods produce valid natural frequencies for a freely vibrating circular plate.

## 2 Miscellaneous Information

Prior to his acceptance in the School of Engineering at the University of Virginia, Dr. Fergusson obtained a Master of Arts degree from the Department of Mathematics at the same university. His grades during his graduate school tenure amounted to an average of B+. No patents were applied for during this period.

## 3 Post-Doctoral Position

A post-doctoral position is currently being occupied by Dr. Fergusson at Tsinghua University in Beijing, The People's Republic of China. The program, which will last for six months starting in August of 1991, is part of a cultural exchange program between Tsinghua University and the University of Virginia.

While on the exchange program at Tsinghua University, Dr. Fergusson is working on the finite dynamic element as applied to the analysis of energy-storing prosthetic feet. In particular, a high-order modal analysis scheme developed by Dr. Fergusson is being employed in the analysis of the forced deformation of a composite prosthetic foot constructed at the Department of Rehabilitation Engineering Medicine at Tsinghua University.

In addition, initial analysis is being accomplished for the construction of a frequency-dependent lumped mass matrix to be used in the dynamic element analysis of a rotating shaft.

# 4 Future work

As a continued part of his technical career, Neil J. Fergusson intends to participate in a one year post-doctoral position at The Engineering Laboratory (MEL) in the Agency of Industrial Science and Technology, Ministry of International Trade and Industry, located in Tsukuba, Japan. During this period, Dr. Fergusson will work on active noise control of a structure-borne sound, and active wave control of a flexible structure.

Upon returning from Japan, Dr. Fergusson hopes to obtain a tenure-track position at a suitable university in the United States, and continue researching in the field of Structural Dynamics.